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Final Report for "Non-Conforming Finite Elements; Mesh Generation, Adaptivity and Related Algebraic Multigrid and Domain Decomposition Methods in Massively Parallel Computing Environment.

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Final Report

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Final Report for "NON–CONFORMING FINITE ELEMENTS; MESH GENERATION, ADAPTIVITY AND RELATED ALGEBRAIC MULTIGRID AND DOMAIN DECOMPOSITION METHODS IN MASSIVELY PARALLEL COMPUTING ENVIRONMENT"

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Title NON–CONFORMING FINITE ELEMENTS; MESH GENERATION, ADAPTIVITY AND RELATED ALGEBRAIC MULTIGRID AND DOMAIN DECOMPOSITION METHODS IN MASSIVELY PARALLEL COMPUTING ENVIRONMENT

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FINAL REPORT ON THE CONTRACT

NON–CONFORMING FINITE ELEMENTS; MESH GENERATION, ADAPTIVITY AND RELATED ALGEBRAIC MULTIGRID AND DOMAIN DECOMPOSITION METHODS IN MASSIVELY PARALLEL COMPUTING ENVIRONMENT

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ABSTRACT. Construction, analysis and numerical testing of efficient solution techniques for solving elliptic PDEs that allow for parallel implementation have been the focus of the research. A number of discretization and solution methods for solving second order elliptic problems that include mortar and penalty approximations and domain decomposition methods for finite elements and finite volumes have been investigated and analyzed. Techniques for parallel domain decomposition algorithms in the framework of PETC and HYPRE have been studied and tested. Hierarchical parallel grid refinement and adaptive solution methods have been implemented and tested on various model problems. A parallel code implementing the mortar method with algebraically constructed multiplier spaces was developed.

1. INTRODUCTION

The discretization methods of PDEs on non-matching grids provides greater flexibility in the grid generation process, increases the portability of various approximation methods and computer implementations, enhances the capabilities of coarsening strategy in parallel algebraic multigrid methods, and provides a natural and practical way for parallel domain decomposition methods and parallel adaptive methods based on a posteriori error analysis.

In the period May 2001 - February 2002 our research focused on the analysis, implementation, and testing of some discretization methods of PDEs on non-matching grids. This approach seems to lead to competitive algorithms that can be used in various codes for complex applications in physics and engineering.

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Key words and phrases. discretization methods, non–matching grids, multilevel methods, iterative methods, domain decomposition.

2. Domain decomposition methods for non-mortar approximations on non-matching grids

The interior penalty method aims at eliminating the need for additional (Lagrange multiplier or mortar) spaces and imposes (only approximately) the required continuity across the interfaces by an appropriate penalty term (completed in [3]). In our approach, the jumps in the values of the functions along these interfaces is penalized in the variational formulation. For smooth solutions we lose the optimal accuracy due to lower approximation at the interface, but on the other hand we produce symmetric and positive definite discrete problems which have optimal condition number.

We also address the issue of constructing preconditioners for solving the system on the composite non–matching grids. We proposed and investigated an interface domain decomposition type preconditioner, that is spectrally equivalent to the reduced (on the interface) algebraic problem. We have tested both the accuracy of the method and the preconditioning technique on a series of model problems.

In [5] we were able to prove an almost optimal error estimate for the interior penalty approximation of the original problem based on the partition of the domain into a finite number of subdomains. Further, an improved error analysis for the finite element approximation of the penalty formulation was derived. Finally, numerical experiments on a series of model second order problems were performed in order to test and verify computationally our theoretical findings.

3. Mortar approximations of finite element and finite volume methods on non-matching grids

We considered an algebraic extension of the local construction for the mortar multipliers based on the general 3–d dual finite element basis described in a paper by Kim, Lazarov, Pasciak and Vassilevski [2]. The purely algebraic construction of the mortar interpolation in the present case requires the inversion of local mass matrices on the non-mortar interface. Since it is purely algebraic, it can be applied to the generalized objects (elements, faces and degrees of freedom) produced by the subdomain AMGe.

A parallel code implementing the mortar method with algebraically constructed multiplier spaces was developed. The target application of this code is in the construction of parallel (including multigrid) preconditioners using element based (AMGe) coarsening in each subdomain. The work was carried out in close collaboration with our summer student T. Kolev from Texas A & M University and Dr. Panayot Vassilevski from CASC, LLNL.

A general code was implemented to illustrate the behavior of the proposed method. It requires input data for each subdomain that includes the element topology, the local subdomain stiffness matrices as well as the mass matrices on the interfaces. This information is independent of the dimension and structure of the problem and is regenerated after an AMGe coarsening.

4. Multilevel adaptive grid refinement and error control

We worked on a problem of multilevel grid refinement and error control for both finite volume approximations and penalty domain decomposition methods. This work was in direct connection with research in CASC on developing and testing of parallel algorithms by J. Jones, V. Henson, R. Falgout, U. Meier, C. Tong, and P. Vassilevski. We have developed 2-D and 3-D codes for parallel adaptive grid refinement that produces nested (and matching) grids. In a closed collaboration with Dr. Panayot Vassilevski and Dr. Charles Tong, the new software was connected to the HYPRE Preconditioner Library.

A parallel mesh generation tool, called ParaGrid, was further developed. The development was a continuation of a 2-D project that was started last summer in CASC. ParaGrid is software that takes as input a coarse tetrahedral mesh, which describes suitably the domain, splits it using METIS, distributes the partitioning among the available processors and generates, in parallel, a sequence of meshes. It has internal solvers and is able to generate various Finite Element/Volume discretizations. The data structures allow ParaGrid to be easily connected to (or used to provide data to) external parallel finite element/volume solvers based on domain decomposition. Generation and solution routines for elasticity problems were added to the code. HYPRE preconditioners and solvers can be used. The connection is done through FEI 3.0. It has been successfully used by several researchers in CASC for algorithm testing purposes.

The software was developed in close collaboration with our student S. Tomov from Texas A&M University and Dr. Charles Tong from CASC. It was used for testing various ideas and strategies in the a posteriori error analysis and error control for convection-diffusion-reaction problems in 3-D domains with complex structure.

5. Interaction with CASC members, visitors, and summer students

Raytcho Lazarov and Joseph Pasciak visited CASC for the month of July, 2000. They were also in a close contact with J. Jones, V. Henson, R. Falgout, C. Tong, and P. Vassilevski and interacted with other long and short term visitors R. Bank, M. Holst, T. Manteuffel, S. McCormick, and G. Douglas, as well as with the summer students P. Dostert, A. Iontcheva, T. Kolev, S. Tomov, and M. Flanagan. Our scientific collaboration for this year resulted in three joint papers that are submitted or already published.

Four Ph. D. students from the program of computational mathematics at Texas A&M University, P. Dostert, M. Flanagan, T. Kolev and S. Tomov, spent two months at CASC LLNL on a professional summer internship and training. Two TAMU Ph. D. students, T. Kolev and S. Tomov, participated directly in the work of the contract.

6. Further collaboration between TAMU and CASC

The Numerical Analysis group at TAMU has expanded and deepened the collaboration with CASC LLNL. The project integrated and enhanced the efforts of TAMU and CASC groups in construction, analysis and numerical testing of efficient, scalar and parallel, discretization techniques and adaptive grid refinement for solving 3-D problems.

The software that was developed for adaptive parallel grid generation (connected to HYPRE Preconditioner Library) will be used as an environment for large scale computations by the Numerical Analysis Group at Texas A&M University, which includes several faculty members and more than 10 Ph. D. students.

We plan to develop efficient parallelizable, scalable algorithms for treating both the geometrically refining spaces or algebraically coarsening grids. The future research should offer alternatives to the existing efforts in CASC for parallel AMG(e) method (based on the ideas from [1]).

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